Kosinussatz

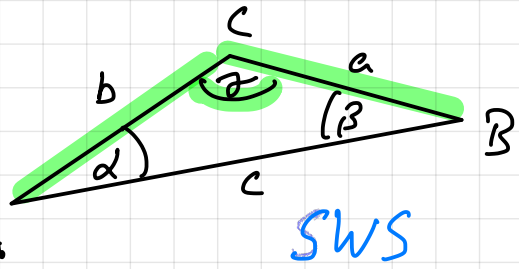
25.3.

gilt in allen Dreiecken

$$a^2 = b^2 + c^2 - 2bc \cdot \cos(\alpha)$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos(\beta)$$

$$c^2 = a^2 + b^2 - 2ab \cos(\gamma)$$



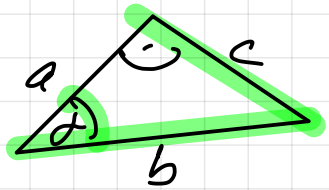
$c^2 = a^2 + b^2$ wenn $\gamma = 90°$ – Satz des Pythagoras

denn $\cos(90°) = 0$

Bsp.: Hypotenuse $b = 8\,cm$

$c = 6\,cm$

$\alpha = 30°$



SSW

nicht Ssw

Berechne fehlende Größen

$$\sin(\gamma) = \frac{c}{b} = \frac{6}{8}$$

$$\gamma = \sin^{-1}\left(\frac{3}{4}\right) = 48{,}6°$$

aber Winkelsumme

$180° \neq 90° + 30° + 48{,}6°$

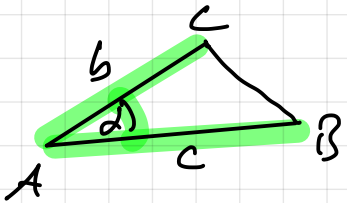
geht nicht

Bsp.:

$\alpha = 30°$

$b = 7\,cm$

$c = 5\,cm$



SWS

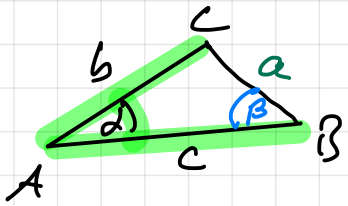
Kongruenzsatz!

Bsp.:

$\alpha = 30°$

$b = 7\,cm$

$c = 5\,cm$



SWS

Kongruenzsatz!

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$a = \sqrt{7^2 + 5^2 - 2 \cdot 5 \cdot 7 \cdot \cos(30°)} = 3{,}7\,cm$$

gesucht $\beta°$

Gleichungen lösen!!!

$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos(\beta) \quad | -a^2 - c^2$$

$$b^2 - a^2 - c^2 = -2 \cdot a \cdot c \cdot \cos(\beta) \quad | \ : (-2ac)$$

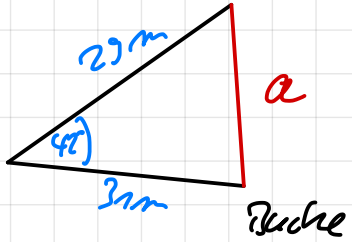
$$\cos(\beta) = \frac{b^2 - a^2 - c^2}{-2ac} \quad | \cos^{-1}()$$

$$\beta = \cos^{-1}\left(\frac{b^2 - a^2 - c^2}{-2ac}\right) \approx 106°$$

$$\gamma = 180° - 106° - 30° = 44°$$

S. 151 Beispiele 1 + 2 !

S. 152 A2 (Planskizze)



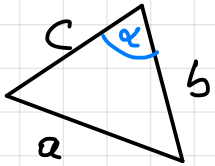
Kosinussatz

$a^2 = 29^2 + 31^2 - 2 \cdot 29 \cdot 31 \cdot \cos(42°)$

$a = \sqrt{465,83} = 21,6$

Der Abstand beträgt 21,6 m.

3a) Kosinussatz selber umformen



α ?

$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$

$10^2 = 12^2 + 11^2 - 2 \cdot 12 \cdot 11 \cdot \cos(\alpha) \quad | -12^2 - 11^2$

$-165 = -264 \cdot \cos(\alpha) \quad | : -264$

$0,625 = \cos(\alpha) \quad | \cos^{-1}()$

$\alpha = \cos^{-1}(0,625) = 51,3°$ ... $\beta = 69,5°$ $\gamma = 59,2°$

S.152 A8 + A14a